Principle of Transmissibility of Force

It states "A force being a sliding vector continues to act along its line of action and therefore makes no change if it acts from a different point on its line of action on a rigid body". Consider a rigid body as shown in Fig. 2.10, acted upon by a force of magnitude F acting at A. The effect on the body would remain unchanged if it acted from point B, C or any other point on its line of action.

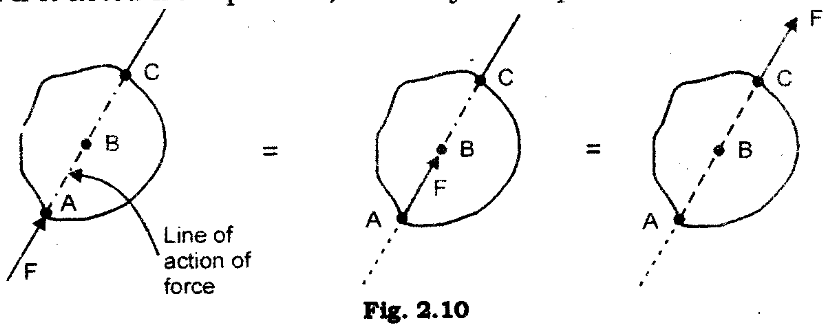


Fig. 2.10

Varignon's Theorem

Varignon, a French mathematician (1654 – 1722) established that the sum of the moments of a concurrent system of forces about any point is equal to the moment of the resultant of the concurrent system about the same point. Though originally derived for a concurrent system of forces, this theorem can in fact be applied to any system of forces and is thus stated as *"the algebraic sum of the moments of a system of coplanar forces about any point in the plane is equal to the moment of the resultant force of the system about the same point"*.

Mathematically it is written as

$$\sum M_A^F = M_A^R \qquad \text{............ 2.2}$$

Sum of moments of all forces about any point, say point A. = Moment of the resultant about the same point A.

Proof – Let P and Q be two concurrent forces at O, making angle θ_1 and θ_2 with the x-axis, let R be their resultant making an angle θ with x-axis.
Let A be a point on the y-axis about which we shall find the moments of P and Q and also of the resultant R. Let d_1, d_2 and d be the moment arm of P, Q and R from moment centre A.

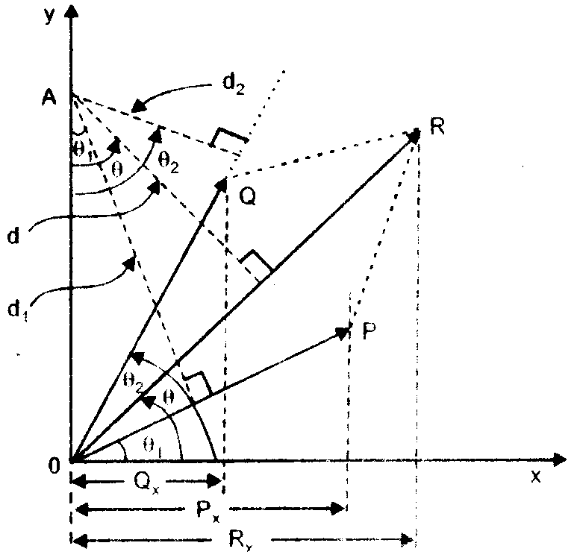


Fig. 2.12

Now,

$$\text{Moment of P about A} = M_A^P = P \times d_1 \qquad \text{---------(1)}$$

$$\text{Moment of Q about A} = M_A^Q = Q \times d_2 \qquad \text{---------(2)}$$

$$\text{Moment of R about A} = M_A^R = R \times d$$
$$= R\,(\,OA \cos\theta\,)$$
$$= OA\,(\,R_x\,) \qquad \text{---------(3)}$$

Adding equations (1) and (2) we have

$$M_A^P + M_A^Q = P\,d_1 + Q\,d_2$$

or sum of moments ΣM_A^F $= P \times OA \cos\theta_1 + Q \times OA \cos\theta_2$

$= OA.\,P_x + OA.Q_x$ since $P_x = P\cos\theta_1$ and $Q_x = Q\cos\theta_2$

$= OA\,(P_x + Q_x)$

$\therefore$ $\Sigma M_A^F = OA\,(R_x)$ ---------(4) since the resultant of forces in the 'x' direction is the sum of components of forces in the 'x' direction

Comparing equation (4) with (3)

$\Sigma M_A^F = M_A^R$ ----- Proved

The above equation can similarly be extended for more than two forces in the system.

Lami's Theorem

Lami's theorem deals with a particular case of equilibrium involving three forces only. It states "*If three concurrent forces act on a body keeping it in equilibrium, then each force is proportional to the sine of the angle between the other two forces*".

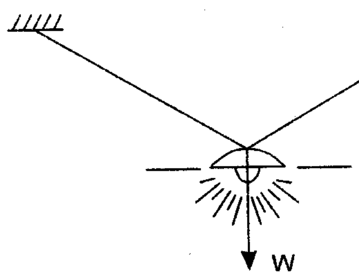


Fig. 3.17 (a)

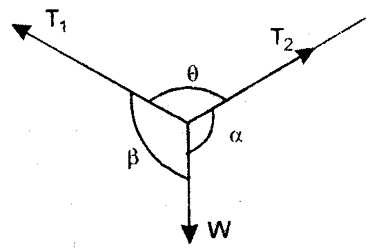


Fig. 3.17 (b)

Fig. 3.17 (a) shows a lamp held by two cables. The two tensile forces T_1 and T_2 in the string and the weight W of the lamp form a system of three forces in equilibrium. The forces would form a concurrent system. If α is the angle between T_2 and W, β is the angle between T_1 and W and θ is the angle between T_1 and T_2 then according to Lami's theorem

$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{W}{\sin\theta}$$

or in general for a system of three forces P, Q and R as shown in Fig. 3.17 (c) we write Lami's equation as

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\theta}$$

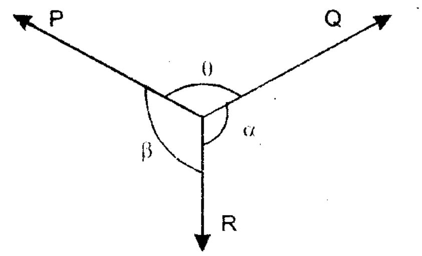


Fig. 3.17 (c)

Note that while using Lami's theorem, the three concurrent forces should either act towards the point of concurrence or act away from it. If this is not the case then using the principle of transmissibility they can be made in the required form. Fig. 3.18 (a) shows such a case for a sphere resting against smooth surfaces. The reactions R_A and R_B act ⊥ to smooth surfaces. To apply Lami's equation the forces have been arranged acting away from the point of concurrence as shown in Fig. 3.18 (b).

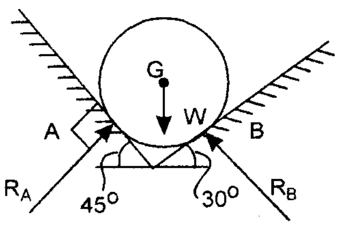


Fig. 3.18 (a)

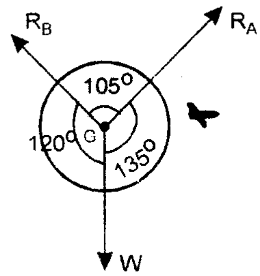


Fig. 3.18 (b)

Proof: Let P, Q and R be the three concurrent forces in equilibrium as shown in Fig. 3.19 (a).

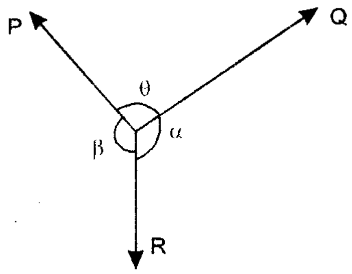


Fig. 3.19 (a)

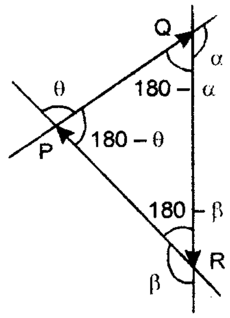


Fig. 3.19 (b)

Since the forces are vectors they are added vectorially by head and tail connections. We get a closed triangular polygon as shown in Fig. 3.19 (b).

Applying sine rule we get

$$\frac{P}{\sin(180 - \alpha)} = \frac{Q}{\sin(180 - \beta)} = \frac{R}{\sin(180 - \theta)}$$

$$\therefore \quad \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\theta} \quad \text{............. proved}$$

Coefficient of Friction

It has been experimentally found that the ratio of the limiting frictional force F_{max} and the normal reaction N is a constant. This constant is referred to as *coefficient of static friction*, denoted as μ_s.

$$\mu_s = \frac{F_{max}}{N} \quad \text{......... } \mathbf{4.1}$$

Similarly the ratio of kinetic frictional force and the normal reaction is known as *coefficient of kinetic friction*, denoted as μ_k.

$$\mu_s = \frac{F_k}{N} \quad \text{......... } \mathbf{4.2}$$

Since F_k is less than F_{max}, we have $\mu_k < \mu_s$

Coefficient of friction μ_s or μ_k depend on the nature of surfaces of contact and have value less than 1. For example the approximate value of coefficient of static friction (μ_s) between wood on glass ranges between 0.2 to 0.6. The corresponding coefficient of kinetic friction (μ_k) is around 25 % lower.

Laws of Friction

1. The frictional force is always tangential to the contact surface and acts opposite to the direction of impending motion.
2. The value of frictional force F increases as the applied disturbing force increases till it reaches the limiting value F_{max}. At this limiting stage the body is on the verge of motion.

3. The ratio of limiting frictional force F_{max} and the normal reaction N is a constant and it is referred to as coefficient of static friction (μ_s).

4. For bodies in motion, frictional force developed (F_k) is less than the limiting frictional force (F_{max}). The ratio of F_k and the normal reaction N is a constant and is referred to as coefficient of kinetic friction (μ_k).

5. The frictional force F generated between the two rubbing surfaces is independent of the area of contact.

Angle of Friction, Cone of Friction and Angle of Repose

Angle of Friction:

"It is the angle made by the resultant of the limiting frictional force F_{max} and the normal reaction N with the normal reaction". Fig. 4.3 shows a block of weight W under the action of the applied force P. Let N be the normal reaction. If the block is on the verge of impending motion, the frictional force F_{max} would be developed as shown.

Let R be the resultant of F_{max} and N, making an angle ϕ with the normal reaction. Here ϕ is known as the *angle of friction.*

Here $R = \sqrt{F_{max}^2 + N^2}$

$= \sqrt{(\mu_s N)^2 + N^2}$

also $\tan\phi = \frac{F_{max}}{N} = \frac{\mu_s N}{N}$

$\tan\phi = \mu_s$

$$\therefore \quad \phi = \tan^{-1}\mu_s \qquad \text{......... 4.3}$$

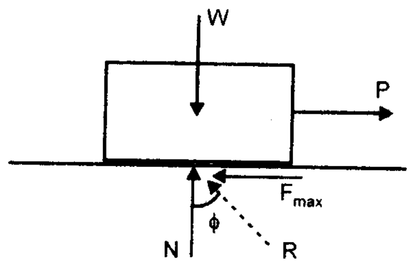


Fig. 4.3

Cone of Friction:

Fig. 4.4 shows a block of weight W on the verge of motion acted upon by force P. Let R be the resultant reaction at the contact surface acting at an angle of friction ϕ. If the direction of force is changed by rotating it through 360° in a plane parallel to the contact surface, the force R also rotates and generates a right circular cone of semi-central angle equal to ϕ. This right circular cone is known as the *cone of friction.* For a body to be stationary the reaction R should be within the cone of friction.

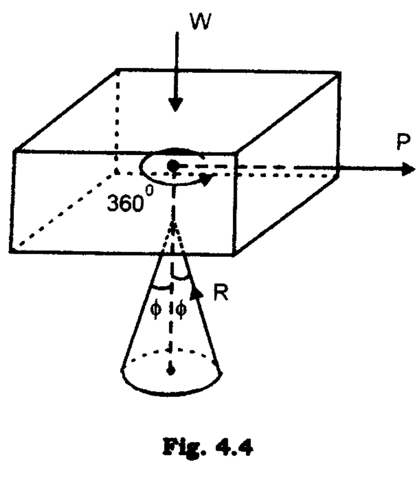


Fig. 4.4

Angle of Repose:

It is defined as the minimum angle of inclination of a plane with the horizontal for which a body kept on it will just slide on it without the application of any external force.

Consider a block of weight W resting on a rough horizontal plane. The plane is slowly tilted till the block is just on the verge of sliding down the plane, Fig. 4.5. The angle of inclination of the plane at this position is known as the *angle of repose*. It is denoted by letter α. Angle of repose is independent of the weight of the body and depends only on the coefficient of static friction. Let us derive the relation between angle of repose α and coefficient of static friction μ_s.

Applying COE

$\sum F_x = 0$ ↗ + ve

$F_{max} - W \sin\alpha = 0$

$\mu_s N - W \sin\alpha = 0$ --------- (1)

$\sum F_y = 0$ ↖ + ve

$N - W \cos\alpha = 0$

$N = W \cos\alpha$ --------- (2)

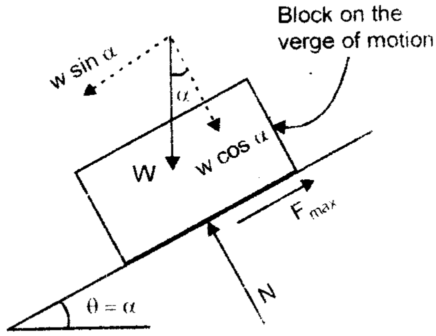


Fig. 4.5

Substituting (2) in (1)

$\mu_s(W \cos\alpha) - W \sin\alpha = 0$

$\tan\alpha = \mu$

or $\alpha = \tan^{-1} \mu_s$ **4.4**

but we have seen that

$\phi = \tan^{-1} \mu_s$

$\therefore \alpha = \phi$

i.e. Angle of Repose = Angle of Friction

Though magnitude wise both angle of repose and angle of friction have the same value, their meaning and application is different, as we have seen.

Analysis of Truss

Truss analysis involves calculation of the support reactions and then finding the axial forces in various members of the truss. The nature of axial forces i.e tensile or compressive is also found out. After the analysis of truss, the design of members is carried out which involves deciding the best suited cross-section of the member and the corresponding cross-sectional area required. Before we study the various methods of truss analysis, let us discuss the assumptions on which the analysis would be based.

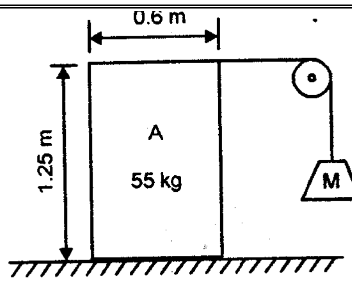
1. All the members of the truss lie in one plane forming what is known as a *plane truss*. Various plane trusses joined together form a *space truss*.
2. All the loads acting on the truss lie in the plane of the truss.
3. The members of the truss are joined at the ends by internal hinges known as pins.
4. Loads act only at the joints and not directly on the members.
5. Each member is a two force body thereby resulting in axial forces which are either tensile or compressive.
6. The self weight of the members being small as compared to the loads, is neglected.
7. The truss is statically determinate i.e. forces can be determined using equilibrium conditions.

There are three method of truss analysis

1. Method of Joints
2. Method of Sections
3. Graphical Method

The Graphical Method of truss analysis has been dealt separately in Chapter 10 of Graphic Statics.

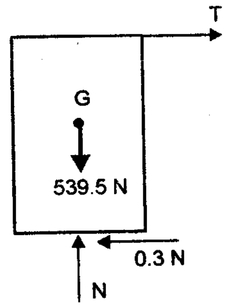
Ex. 4.15 What value of mass M will cause motion of block A to impend. $\mu = 0.3$ between A and plane and $2/\pi$ between rope and drum.



Solution: Since the height of application of the force is given, there are two possibilities.

1st possibility: The block slides.

Isolating the block. Let T be the tension in the connecting portion of the rope.



Applying COE to the block

$$\Sigma F_y = 0 \uparrow + ve$$

$$N - 539.5 = 0$$

$$N = 539.5 \text{ N}$$

$$\Sigma F_x = 0 \rightarrow + ve$$

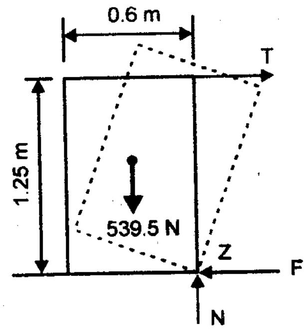
$$T - 0.3\,N = 0$$

$$T - 0.3 \times 539.5 = 0$$

$$T = 161.86 \text{ N}$$

2nd possibility: The block tips.

Let the block just tip about the corner Z.



Applying COE to the block

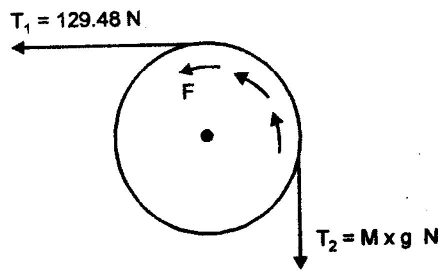
$$\Sigma M_Z = 0 \quad \circlearrowleft + ve$$

$$-T \times 1.25 + 539.5 \times 0.3 = 0$$

$$T = 129.48 \text{ N}$$

Since the block requires 161.86 N to slide and 129.48 N to tip, tipping action would initiate the motion of the block first.

Figure shows the FBD of the drum. Tension on the one side of the rope is 129.48 N and on the other end is due to weight of mass M kg.



Using rope friction formula

$$T_2 = T_1\, e^{\mu\beta}$$

$$M \times 9.81 = 129.48\; e^{(2/\pi)\times(\pi/2)}$$

$$M = 35.87 \text{ kg}$$ ....**Ans.**